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UNIVERSITY EXAMINATIONS 2019/2020

THIRD YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER

SMA 3350: GROUP THEORY

DATE: JANUARY 2021 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question* ***one*** *and any other* ***two*** questions.

QUESTION ONE (30 MARKS)

a) Express the following permutations as a product of disjoint cycles;

i) $\begin{pmatrix} 1\,2\,3\,4\,5\,6\,7\,8\,9 \\ 2\,3\,4\,5\,1\,6\,7\,9\,8 \end{pmatrix}$ **(2 marks)**

ii) $\begin{pmatrix} 1\;a\;b\;c\;d\;e \\ b\;1\;a\;e\;c\;d \end{pmatrix}$ **(3 marks)**

b) Let G be the group of all real 2×2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ with $ad - bc \neq 0$ under matrix multiplication, and let $K = \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}$. Show that K is a subgroup of G. **(4 marks)**

c) i) If H is a subgroup of G and N is a normal subgroup of G, show that $H \cap N$ is a normal subgroup of H. **(5 marks)**

iii) Show that every subgroup of an abelian group is normal. **(3 marks)**

d) In each of the following, verify if the mappings defined are homomorphisms, and in those cases in which they are homomorphisms determine the kernel;

i) $\emptyset : G \to G$ where G is the group of non-zero real numbers under multiplication and $\emptyset$ is defined by $\emptyset(x) = x^2$, $\forall\, x \in G$. **(4 marks)**

ii) $\emptyset : G \to G$ where G is as in (i) and $\emptyset(x) = 2^x$, $\forall\, x \in G$. **(2 marks)**

iii) $\emptyset : G \to G$ where G is the group of real numbers under addition and $\emptyset(x) = x + 1$, $\forall\; x \in G$. **(2 marks)**

e) Write down the multiplication table for S_3. Determine the centre of S_3. **(5 marks)**

QUESTION TWO (20 MARKS

a) State and prove the Lagrange's theorem. **(8 marks)**

b) Let $H = \{I, (12)\}$ be a subgroup of S_3. Find all the left cosets of H in S_3. Determine the index of H in S_3. **(6 marks)**

c) Prove that the order of every element of a finite group is a divisor of the order of a group. **(6 marks)**

QUESTION THREE (20 MARKS)

a) Prove that H is a normal subgroup of G iff $\forall\ x \in G,\ \mathrm{h} \in H\ , x\mathrm{h}x^{-1} \in H$.**(6 marks)**

b) If H and N are subgroups of a group G and given that $H \leq K$, show that $i(G/H) = i(G/K) \times i(K/H)$ **(6 marks)**

c) Given that G is a group;

i) Define the centre, Z, of G. **(2 marks)**

ii) Show that the centre, Z, of a group G, is always a normal subgroup of the group. **(6 marks)**

QUESTION FOUR (20 MARKS)

a) Determine which of the following permutations are even;

i) $(123)(12)$ **(3 marks)**

ii) $(12345)(123)(45)$ **(3 marks)**

iii) $(12)(13)(14)(25)$ **(3 marks)**

b) Prove that the 4 permutations $I, (ab), (cd)$ and $(ab)^{\circ}(cd)$ on 4 symbols a, b, c and d form a finite abelian group with respect to the composite operation. **(11 marks)**

QUESTION FIVE (20 MARKS)

a) Define the following terms:

i) Homomorphism. **(2 marks)**

ii) Isomorphism **(2 marks)**

b) Show that the additive group $(Z, +)$ of integers is isomorphic to the additive group $(G = \{ma : a \in Z\}, +)$. **(6 marks)**

c) Let G and G' be two isomorphic groups whose composition is defined multiplicatively and let f be the corresponding isomorphism. Prove that:

i) f maps the identity element $e \in G$ onto $e' \in G'$, i.e $f(e) = e'$. **(5 marks)**

ii) f maps the inverse of element $a \in G$ onto the inverse of $a \in G'$ i.e $f(a^{-1}) = (f(a))^{-1}, \forall\ a \in G$. **(5 marks)**