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UNIVERSITY EXAMINATIONS 2019/2020

THIRD YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF COMPUTER SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

SMA 3353: RING THEORY

DATE: JANUARY 2021 **TIME: 3 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two*** questions.

QUESTION ONE (30 MARKS)

a) Define the term unit group of a ring $(R, +, .)$. (2 marks)

b) Determine the unit group of the ring $(Z, +, .)$ of integers. (3 marks)

c) i) Define the left ideal of a ring . (2 marks)

ii) Prove that in a ring M_2 of all 2×2 matrices over integers, the set $U = \left\{\begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix} : a, b \in Z\right\}$ is a left ideal but not a right ideal.

(6 marks)

d) i) Define homomorphism of rings. (3 marks)

ii) Prove that the function $f : (Z_4, +_4, \cdot_4) \to (Z_2, +_2, \cdot_2)$ defined by $f(0) = f(2) = 0$ and $f(1) = f(3) = 1$ is a homomorphism. (5 marks)

e) Show that the polynomial $f(x) = x^3 + x + 1$ is irreducible over the field F of integers modulo 5 i.e $(F, +_5, \cdot_5)$.

(5 marks)

f) Find all the zero divisors of Z_{20} . (4 marks)

QUESTION TWO: (20 MARKS)

a) i) Define a subring of a ring R. (3 marks)

ii) Let $s = \{a + b\sqrt{3} : a, b \in Z\}$. Show that S is a subring of the ring R of real numbers. (6 marks)

b) i)Let $(R, +,\cdot)$ be a ring and $(I, +,\cdot)$ be an ideal of the ring $(R, +,\cdot)$. Prove that $(I, +,\cdot)$ is a subring of $(R, +,\cdot)$. (6 marks)

ii) Using the ring $(Q, +,\cdot)$, of rational numbers and the subring $(Z, +,\cdot)$ of $(Q, +,\cdot)$, prove that a subring is not necessarily an ideal of the ring. (5 marks)

QUESTION THREE: (20 MARKS)

a) Find the greatest common divisor of the following polynomials over F, the field of rational numbers: $x^2 + x - 2$ and $x^5 - x^4 - 10x^3 + 10x^2 + 9x - 9$. (7 marks)

b) Prove that $x^3 - 9$ is irreducible over the field of integers modulo 31. (10 marks)

c) Prove that the polynomial $f(x) = 1 + x + x^3 + x^4$ is not irreducible over any field F.

(3 marks)

QUESTION FOUR: (20 MARKS)

a) Explain the following terms as used in ring theory;

i) Binary operation. (2 marks)

ii) Ring. (2 marks)

iii) Integral domain. (2 mark)

iv) Quotient ring. (2 mark)

v) Zero divisor of a ring. (2 marks)

b) Prove that the intersection of two subrings of a ring R is a subring of R. (5 marks)

c) Let $(Z, +,\cdot)$ be a ring of integers and $(5Z, +,\cdot)$ be an ideal generated by 5. Prove that the system $\left({}^{Z}/_{5Z}, +,\cdot\right)$ is a quotient ring. (5 marks)

QUESTION FIVE (20 MARKS)

a) Prove that the set Z of integers is a ring with respect to addition and multiplication.

(10 marks)

b) Prove that the $(\{0,1,2,3,4,5\}, +_6, \times_6)$ is a ring with zero divisors.

(5 marks)

c) If R is a commutative ring and $a \in R$, show that $aR = \{ar : r \in R\}$ is a two sided ideal of R.

(5 marks)