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UNIVERSITY EXAMINATIONS 2021/2022

FOURTH YEAR, SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

SMA 3451: TOPOLOGY II

DATE: JANUARY 2022 **TIME: 2 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) i. Define a base for a topology τ. (2 Marks)

ii. Let $X = \{a,b,c\}$ and $\tau = p(x)$ (the power set of x). Determine the base for the topology τ.

(3 Marks)

b) i. Define a compact subset of a topological space X. (2 Marks)

ii. Given that $A = (0,1)$ is an open interval of the real line $\mathbb{R}$. Prove that A is not compact in $\mathbb{R}$. (4 Marks)

c) i. Distinguish between a accountably compact subset and sequentially compact subset of a topological space X. (4 Marks)

ii. Prove that every bounded closed interval $A = [a,b]$ is accountably compact.

(4 Marks)

d) prove that the class of open intervals $A = \left\{(2,3), \left(2,\frac{3}{2}\right), (2,1) \left(2,\frac{3}{4}\right), \left(2,\frac{3}{5}\right), \ldots\right\}$ satisfies the finite intersection property. (5 Marks)

e) Let $A = (0,1), B(1,2)$ and $C = (2,8)$ be interval on $\mathbb{R}$. Prove that

i. A and B are separated sets. (3 Marks)

ii. B and C are not separated. (3 Marks)

QUESTION TWO (20 MARKS)

a) i. Define a disconnected subset A of a topological space X. (2 Marks)

iii. Consider the following topology on $X = \{a, b, c, d, d, e\}$ and $\mathcal{r} = \{X, \emptyset, \{a, b, c\}, \{c, d, e\}\}$.

Show that the subject $A = \{a, d, e\}$ of X is disconnected. (6 Marks)

b) Let X be a topological space. Prove that X is disconnected if there exists a non-empty proper subset of X which is both open and closed. (8 Marks)

c) Given that $X = \{a, b, c, d, e\}$ and$T = \{X, \emptyset, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}\}$. Determine the components of X. (4 Marks)

QUESTION THREE (20 MARKS)

a) Explain the term arcwise connected set. (3 Marks)

b) Prove that continuous image of arcwise connected. (10 Marks)

c) Show that a continuous image of a sequentially compact. (7 Marks)

QUESTION FOUR (20 MARKS)

a) Explain the terms;

i. Simply connected spaces. (2 Marks)

ii. Homotopic spaces (2 Marks)

iii. Disconnection of a set (2 Marks)

b) Given that A and B are connected sets which are not separated, prove that AUB is connected. (8 Marks)

c) Let $X = \{a, b, c, d, e\}$ and $\tau = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}\}$ to be a topology on X.

i. Determine the disconnection of X

ii. The relative topology on the subset$A = \{b, d, e\}$. (4 Marks)

QUESTION FIVE (20 MARKS)

a) Define the terms;

i. Product topology for topological spaces x and y. (3 Marks)

ii. Basic neighborhoods of$(x, y)\epsilon XxY$. (2 Marks)

b) Consider the topologies $\tau_1 = \{X, \phi, \{d\}, \{b, c\}\}$ on $X = \{b, c, d\}$ and $\tau_2 = \{Y, \phi, \{u\}\}$ of $Y = \{u, v\}$

i. Determine the defining subbase of the product topology on XxY. (5 Marks)

ii. Determine the defining base for the product topology on Xxy. (5 Marks)

c) Prove that a finite subset of a topological space X is sequentially compact. (5 Marks)